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# Dyon Solutions in Non-Temporal $SU(3)\otimes SU(3)$ Gauge

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**Abstract** - Employing the Cabbibo–Ferrari type non- Abelian field tensor we consider the $SU(3)\otimes SU(3)$ gauge theory under the non-temporal gauge conditions and show that the obtained solutions are dyonic and have finite energy.

**Index Term:** Dyon Solutions, Non-Abilian Field Tensor, Gauge Field Theory

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## Introduction

In 1930s Dirac[1] advanced the idea that isolated magnetic poles might exist. The idea of magnetic monopoles got a boost in 1970s when 't Hooft[2] and Polyakov[3] showed that in gauge field theories in which the symmetry group is spontaneously broken possess classical solutions with the natural interpretation of magnetic monopoles. Soon the Julia and Zee's[4] conjecture was seen as the non-Abelian analogue of Schwinger's Abelian dyons[5]. The interest on monopoles and dyons generated by Dirac[1], 't Hooft[2], Polyakov[3] and Julia and Zee[4] has remained undiminished and extensive theoretical and experimental works on the related topics have been undertaken[6-21, 30].

Since, the solutions which were interpreted as magnetic monopoles were originally found in SO(3) gauge group and this group being small for unifying electromagnetic and weak interactions, larger gauge groups like SU(3) were explored[8-12, 22, 23] . A key factor of such theories is the twin combination of the choice of gauge and choice of gauge field tensor. Theories have in general followed the approach of Julia and Zee[4] and employed usual Yang-Mills type field tensor and have used temporal gauge conditions to arrive at monopole solutions and obtained dyon solutions in non-temporal gauge.

In 1960s, Cabbibo and Ferrari[24] developed a two potential field tensor for developing a theory of Abelian dyons and Yang Mills type field tensor continued to be used for dyon solutions in non-Abelian gauge theories.

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One of the authors (DCJ) has in earlier papers[11] developed a Cabbibo-Ferrari[24] type field tensor for non-Abelian fields and employed[12-13] it on non-Abelian gauge theories with electric and magnetic sources. Using the same field tensor and the Kyriakopoulos[22] technique we show in the previous paper that the dyon solutions be obtained in the temporal gauge [(31)]. The Kyriakopoulos [(22)] technique under the temporal gauge conditions reduced the gauge field equations into the first order differential equations whose solutions depicted a set of dyon solutions. Extending the analysis in the present paper we examine the $SU(3)\otimes SU(3)$ gauge under the non- temporal gauge conditions and find that in this case too we obtain the finite energy dyon solutions but unlike the previous case they emerge as the solutions of second order differential equations. The paper has been divided into six sections. Section 2 defines the Lagrangian density, the gauge group of the theory, field equations and matrix notation .The ansatz for obtaining the solutions has been presented in section 3. The solutions have been shown to have finite energy in section 4.the adjoining solutions be obtained in section 5. That the obtained solutions belong to electric and magnetic charges has been shown in section 6 to which then follow the concluding remarks.

## 2. The Gauge Group and the Lagrangian Density

In this section we briefly recapitulate the steps from the previous paper[(31)].

The system whose gauge group is $SU(3)\otimes SU(3)$, is described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} \times G^{\mu\nu a} + \frac{1}{2}\left(D_{\mu}\phi\right)^{a} \times \left(D^{\mu}\phi\right)^{a} + V\left(\phi^{a} \times \phi^{a}\right) \quad (1)$$

where[(31)]

$$G^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} - e f^{abc} A^{b}_{\mu} A^{c}_{\nu} - \frac{1}{2} \delta_{\mu\nu\rho\sigma}\left(\partial^{\rho} B^{\sigma a} - \partial^{\sigma} B^{\rho a} - g\ f^{abc} B^{\rho b} B^{\sigma c}\right) \quad (2a)$$

and its dual

is
$$\tilde{G}^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu - g f^{abc} B^b_\mu B^c_\nu + \frac{1}{2}\delta_{\mu\nu\rho\sigma}\left(\partial^\rho A^{\sigma a} - \partial^\sigma A^{\rho a} - e f^{abc} A^{\rho b} A^{\sigma c}\right) \quad (2b)$$

in which gauge fields $A^a_\mu$ and $B^a_\mu$ transform as

$$A_\mu \to U A_\mu U^{-1} - \frac{1}{e}\left(\partial_\mu U\right)U^{-1} \quad (3a)$$

and $$B_\mu \to U B_\mu U^{-1} - \frac{1}{g}\left(\partial_\mu U\right)U^{-1} \quad (3b)$$

where $U$ is a gauge function

$$U = \exp\left(-i\Lambda^a T^a\right) \quad (4)$$

with $\Lambda^a$ the real functions of space-time and $T^a$ representing the group generators of SU(3) group obeying

$$[T^a, T^b] = i f^{abc} T^c \quad (5)$$

The $f^{abc}$ are the SU(3) structure constants with a, b, c running from 1 to 8. $T^a = \frac{\lambda^a}{2}$, where $\lambda^a$ (a = 1, 2, … 8) are eight Gell-Mann matrices[25].

The ($\times$) in the Lagrangian density (1) indicates the products in which the fields have been assumed mutually non-interacting. As a result of this assumption the mutual interaction terms, i.e. the cross-terms, disappear leaving

$$G^a_{\mu\nu} \times G^{\mu\nu a} = A^a_{\mu\nu} A^{\mu\nu a} + \tilde{B}^a_{\mu\nu}\tilde{B}^{\mu\nu a} \quad (6)$$

$$\left(D_\mu\phi\right)^a \times \left(D^\mu\phi\right)^a = \left(D^1_\mu\phi^a_e + D^2_\mu\phi^a_g\right)\times\left(D^{1\mu}\phi^a_e + D^{2\mu}\phi^a_g\right) = D^1_\mu\phi^a_e D^{1\mu}\phi^a_e + D^2_\mu\phi^a_g D^{2\mu}\phi^a_g \quad (7)$$

and $$\phi^a \times \phi^a = \left(\phi^a_e + \phi^a_g\right)\times\left(\phi^a_e + \phi^a_g\right) = \phi^a_e\phi^a_e + \phi^a_g\phi^a_g \quad (8)$$

where

$$A^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e f^{abc} A^b_\mu A^c_\nu \quad (9)$$

and $$\tilde{B}^a_{\mu\nu} = \frac{1}{2}\delta_{\mu\nu\rho\sigma} B^{\rho\sigma a} \quad (10)$$

with

$$B^{\rho\sigma a} = \partial^\rho B^{\sigma a} - \partial^\sigma B^{\rho a} - g f^{abc} B^{\rho b} B^{\sigma c} \quad (11)$$

$$D^1_\mu = \partial_\mu - e f^{abc} A^b_\mu \quad (12)$$

$$D^2_\mu = \partial_\mu - g f^{abc} B^b_\mu \quad (13)$$

and $$\phi^a = \phi^a_e + \phi^a_g \quad (14)$$

The covariant derivative $D_\mu\phi^a$ which expressed as

$$\left(D_\mu\phi\right)^a = D^1_\mu\phi^a_e + D^2_\mu\phi^a_g \quad (15)$$

transform as

$$(D_\mu\phi)^a \to U(D_\mu\phi)^a \quad (16)$$

The potential energy $V\left(\phi^a \times \phi^a\right)$ in the Lagrangian density (1) describe the self interaction of field $\varphi^a$ and has the form

$$V\left(\phi^a \times \phi^a\right) = -\eta\left(\phi^a_e\phi^a_e + \phi^a_g\phi^a_g - \xi^2\right)^2 \quad (17)$$

in which $\eta$ and $\xi$ are real constants with $\eta << 1$. The fields $\varphi^a$ may denote the Higgs[26,27] triplet fields.

The Euler-Lagrange variations of the Lagrangian density (1) with respect to $A^a_\mu$, $B^a_\mu$, $\phi^a_e$ and $\phi^a_g$ lead to the field equations

$$\partial_\mu A^{\mu\nu a} - e f^{abc} A^b_\mu A^{\mu\nu c} - e f^{abc}\phi^b_e D^{1\nu}\phi^c_e = 0 \quad (18)$$

$$\partial_\mu \tilde{B}^{\mu\nu a} - g f^{abc} B^b_\mu \tilde{B}^{\mu\nu c} - g f^{abc}\phi^b_g D^{2\nu}\phi^c_g = 0 \quad (19)$$

$$\partial_\mu D^{1\mu}\phi^a_e - e f^{abc} A^b_\mu D^{1\mu}\phi^c_e - \frac{\partial V}{\partial\phi^a_e} = 0 \quad (20)$$

and $$\partial_\mu D^{2\mu}\phi^a_g - g f^{abc} B^b_\mu D^{2\mu}\phi^c_g - \frac{\partial V}{\partial\phi^a_g} = 0 \quad (21)$$

Introducing the notation

$$A_\mu = e A^a_\mu T^a \quad (22a)$$

and $$B_\mu = g B^a_\mu T^a \quad (22b)$$

and also express the Higgs field $\varphi$ as

$$\phi = (e+g)\times\phi^a T^a \equiv e\phi^a_e T^a + g\phi^a_g T^a \equiv \phi_e + \phi_g \quad (22c)$$

where $T^a = \frac{\lambda^a}{2}$ with $\lambda^a$ (a = 1,2,…..,8) the Gell-Mann matrices (25), we may express the field equations (18) to (21) in matrix notation as

$$\partial_\mu A^{\mu\nu} + i[A_\mu, A^{\mu\nu}] + i[\phi_e, D^{1\nu}\phi_e] = 0 \quad (23)$$

$$\partial_\mu \tilde{B}^{\mu\nu} + i[B_\mu, \tilde{B}^{\mu\nu}] + i[\phi_g, D^{2\nu}\phi_g] = 0 \quad (24)$$

$$\partial_\mu D^{1\mu}\phi_e + i\left[A_\mu, D^{1\mu}\phi_e\right] - e\frac{\partial V}{\partial\phi^a_e}T^a = 0 \quad (25)$$

$$\partial_\mu D^{2\mu}\phi_g + i\left[B_\mu, D^{2\mu}\phi_g\right] - g\frac{\partial V}{\partial\phi^a_g}T^a = 0 \quad (26)$$

respectively. It is obvious from the above that[11]

### 3. The Ansatz

In the previous paper[31] the gauge field obeyed the temporal gauge conditions and here temporal parts $A_\mu$ and $B_\mu$ do not vanish we were required to have the ansatz [28]

$$\hat{\alpha} = x^1\lambda^7 - x^2\lambda^5 + x^3\lambda^2 = \hat{\alpha}^a\frac{\lambda^a}{2} = \hat{\alpha}^a T^a$$

$$\hat{\beta} = \frac{4}{3}\mathrm{I}r^2 - 2\hat{\alpha}\hat{\alpha}$$

$$= 2\left(x^1x^2\lambda^1 + x^3x^1\lambda^4 + x^2x^3\lambda^6\right) + \left[(x^1)^2 - (x^2)^2\right]\lambda^3 + \left[r^2 - 3(x^3)^2\right]\frac{\lambda^8}{\sqrt{3}} \tag{27}$$

where $r = \left[(x^1)^2 + (x^2)^2 + (x^3)^2\right]^{\frac{1}{2}}$, $x^1$, $x^2$ and $x^3$ being the components of distance three-vector. We also introduce the three-vector functions $\vec{P}, \vec{Q}, \vec{R}, \vec{S}, \vec{T}$ and $\vec{U}$ expressed by[28]

$$\vec{P} = \vec{\nabla}\hat{\alpha} \tag{28}$$

$$\vec{Q} = \frac{1}{2}\vec{\nabla}\hat{\beta} \tag{29}$$

$$\vec{R} = \vec{x}\hat{\alpha} \tag{30}$$

$$\vec{S} = \vec{x}\hat{\beta} \tag{31}$$

$$\vec{T} = -\vec{x}\times\vec{\nabla}\hat{\alpha} \tag{31}$$

$$\vec{U} = -\frac{1}{2}\vec{x}\times\vec{\nabla}\hat{\beta} \tag{32}$$

and[29]

$$\vec{A} = \frac{(1-T_A)}{r^2}\vec{T} - \frac{U_A}{r^3}\vec{U} \tag{33}$$

$$\vec{B} = \frac{(1-T_B)}{r^2}\vec{T} - \frac{U_B}{r^3}\vec{U} \tag{34}$$

$$A_0 = \frac{R_A}{r^2}\hat{\alpha} + \frac{S_A}{r^3}\hat{\beta} \tag{35}$$

and $$B_0 = \frac{R_B}{r^2}\hat{\alpha} + \frac{S_B}{r^3}\hat{\beta} \tag{36}$$

where $T_A, T_B, U_A, U_B$ $R_A, S_B, R_A, S_B$ are purely r dependent.

The ansatz for the Higgs fields $\phi_e + \phi_g = \phi$ as before[28, 29]

$$\phi_e = \frac{N_{\phi_e}}{r^2}\hat{\alpha} + \frac{M_{\phi_e}}{r^3}\hat{\beta} \tag{37}$$

and $$\phi_g = \frac{N_{\phi_g}}{r^2}\hat{\alpha} + \frac{M_{\phi_g}}{r^3}\hat{\beta} \tag{38}$$

where the coefficients N and M too are purely r-dependent. We also introduce the vector

**4. Finite energy Solutions.**

In earlier paper defined[31]

$$\vec{\mathcal{A}} = \left(A^{23}, A^{31}, A^{12}\right) = \tilde{P}_A\vec{P} + \tilde{Q}_A\vec{Q} + \tilde{R}_A\vec{R} + \tilde{S}_A\vec{S} \tag{39}$$

$$\vec{\tilde{\mathcal{B}}} = \left(\tilde{B}^{01}, \tilde{B}^{02}, \tilde{B}^{03}\right) = \left(B^{23}, B^{31}, B^{12}\right) = \tilde{P}_B\vec{P} + \tilde{Q}_B\vec{Q} + \tilde{R}_B\vec{R} + \tilde{S}_B\vec{S} \tag{40}$$

where $\vec{P}, \vec{Q}, \vec{R}, \vec{S}$ have been defined in equations (28) to (30) and

$$\tilde{P}_A = -\frac{T_A'}{r} \tag{41}$$

$$\tilde{P}_A + r^2\tilde{R}_A = \frac{1 - T_A{}^2 - U_A{}^2}{r^2} \tag{42}$$

$$\tilde{Q}_A = -\frac{U_A'}{r^2} \tag{43}$$

$$\tilde{Q}_A + r^2\tilde{S}_A = -\frac{3T_AU_A}{r^3} \tag{44}$$

and

$$\vec{D}_1\phi_e = \tilde{P}_{e4}\vec{P} + \tilde{Q}_{e4}\vec{Q} + \tilde{R}_{e4}\vec{R} + \tilde{S}_{e4}\vec{S} \equiv \mathcal{A}_4 \tag{45}$$

$$\vec{D}_2\phi_g = \tilde{P}_{g4}\vec{P} + \tilde{Q}_{g4}\vec{Q} + \tilde{R}_{g4}\vec{R} + \tilde{S}_{g4}\vec{S} \equiv \mathcal{B}_4 \tag{46}$$

where

$$\tilde{P}_{e4} = \frac{N_{\phi_e}T_A + M_{\phi_e}U_A}{r^2} \tag{47}$$

$$r^2\tilde{R}_{e4} + \tilde{P}_{e4} = \frac{rN_{\phi_e}' - N_{\phi_e}}{r^2} \tag{48}$$

$$\tilde{Q}_{e4} = \frac{N_{\phi_e}U_A + 2M_{\phi_e}T_A}{r^3} \tag{49}$$

$$r^2\tilde{S}_{e4} + \tilde{Q}_{e4} = \frac{rM_{\phi_e}' - M_{\phi_e}}{r^3} \tag{50}$$

with similar relations with $e \to g$ and $A \to B$.

As shown in the following subsection, the ansatz (33), (34), (37) and (38) allow us to write the field equations (18)–(21) in terms of field equations without $SU(3)$ indices.

We use the same ansatz and notations as used in the earlier paper(30) for temporal gauge. We also employ the ansatz for non temporal gauge(22)

we can express the space-time component of $A^{\mu\nu}$ and $\tilde{B}^{\mu\nu}$ as(28)

$$\mathcal{A}_0 = (A_{10}, A_{20}, A_{30}) = \tilde{P}_{0A}\vec{P} + \tilde{Q}_{0A}\vec{Q} + \tilde{R}_{0A}\vec{R} + \tilde{S}_{0A}\vec{S} \quad (51)$$

Where

$$\tilde{P}_{0A} == \frac{R_A T_A + 2S_A U_A}{r^2} \quad (52)$$

$$r^2\tilde{R}_{0A} + \tilde{P}_{0A} = \frac{rR'_A - R_A}{r^2} \quad (53)$$

$$\tilde{Q}_{0A} = \frac{R_A U_A + 2S_A T_A}{r^3} \quad (54)$$

$$r^2\tilde{S}_{0A} + \tilde{Q}_{0A} = \frac{rS'_A - S_A}{r^3} \quad (55)$$

and

$$\begin{aligned}\tilde{\mathcal{B}}_0 &= (\tilde{B}_{23}, \tilde{B}_{13}, \tilde{B}_{12}) = (B_{10}, B_{20}, B_{30}) \\ &= \tilde{P}_{0B}\vec{P} + \tilde{Q}_{0B}\vec{Q} + \tilde{R}_{0B}\vec{R} + \tilde{S}_{0B}\vec{S}\end{aligned} \quad (56)$$

where

$$\tilde{P}_{0B} = \frac{R_B T_B + 2S_B U_B}{r^2} \quad (57)$$

$$r^2\tilde{R}_{0B} + \tilde{P}_{0B} = \frac{rR'_B - R_B}{r^2} \quad (58)$$

$$\tilde{Q}_{0B} = \frac{R_B U_B + 2S_B T_B}{r^3} \quad (59)$$

$$r^2\tilde{S}_{0B} + \tilde{Q}_{0B} = \frac{rS'_B - S_B}{r^3} \quad (60)$$

Now we look at the field equations [31] (23) to(26) and separate their space and time components. Using eqyuations (51) and (56) the respective space and timecomponents of (23) and (24) can jbe expressed as

$$\nabla\times\mathcal{A} + (\vec{A}\times\mathcal{A} + \vec{A}\times\mathcal{A}) - i[A_{0,}\, \vec{\mathcal{A}}_0] + i[\phi_e, \vec{D}^1\phi_e] = 0 \quad (61)$$

$$\nabla\times\mathcal{B} + (\vec{B}\times\mathcal{B} + \vec{A}\times\mathcal{B}) - i[B_{0,}\, \vec{\mathcal{B}}_0] + i[\phi_g, \vec{D}^2\phi_g] = 0 \quad (62)$$

$$\nabla\vec{\mathcal{A}}_0 - i[\vec{A}\,\vec{\mathcal{A}}_0 - \vec{\mathcal{A}}_0\vec{A}] = 0 \quad (63)$$

and $\nabla\tilde{\mathcal{B}}_0 - i[\vec{B}\,\tilde{\mathcal{B}}_0 - \tilde{\mathcal{B}}_0\vec{B}] = 0$ (64)

where $\vec{\mathcal{A}}$ and $\vec{\mathcal{B}}$ are (39) and (40) for the space and time parts of eqs (25)and (250, we observe their V = 0 and find that, due to the static nature of fields and the ansatz (25) and (26) vanish leaving the space parts as

$$\nabla(\vec{D}^1\phi_e) + i\left[\vec{A}, \vec{D}^1\phi_e\right] = 0 \quad (65)$$

$$\nabla(\vec{D}^2\phi_g) + i\left[\vec{B}, \vec{D}^2\phi_g\right] = 0 \quad (66)$$

Now we first look at the set of eqs. (61), (62) and (65) that contain the space parts $\vec{A}$ of the gauge field $A_\mu$. Using eqs (34) in these equations we can calculate the individual terms as

$$\nabla\times\mathcal{A} == \left[\frac{T''_A}{r^2} - \frac{(1 - T_A{}^2 - U_A{}^2)}{r^4}\right]\vec{T} + \left[\frac{U''_A}{r^3} - \frac{6U_A T_A}{r^5}\right]\vec{U} \quad (67a)$$

$$\begin{aligned}+i(\vec{A}\times\mathcal{A} - \vec{A}\times\mathcal{A}) &= \left[\frac{(1-T_A)(1 - T_A{}^2 - U_A{}^2)}{r^4} - \frac{6T_A U_A^2}{r^4}\right]\vec{T} \\ &+ \left[\frac{U_A(1 - T_A{}^2 - U_A{}^2)}{r^5} - \frac{6(1-T_A)U_A T_A}{r^5}\right]\vec{U} \quad (67b)\end{aligned}$$

$$\begin{aligned}-i[A_{0,}\, \vec{\mathcal{A}}_0] &= \left[\frac{T_A(R_A{}^2 + 4S_A{}^2) + U_A R_A S_A}{r^4}\right]\vec{T} \\ &+ \left[\frac{U_A(R_A{}^2 + 4S_A{}^2) + T_A R_A S_A}{r^5}\right]\vec{U}\end{aligned} \quad (67c)$$

$$\begin{aligned}+i[\phi_e, \vec{D}^1\phi_e] &= -\frac{T_A(N_{\phi_e}{}^2 + 4M_{\phi_e}{}^2) + 4U_A N_{\phi_e} M_{\phi_e}}{r^4}\vec{T} \\ &- \frac{U_A(N_{\phi_e}{}^2 + 4M_{\phi_e}{}^2) + 4T_A N_{\phi_e} M_{\phi_e}}{r^5}\vec{U}\end{aligned} \quad (67d)$$

$$\begin{aligned}&\nabla\vec{\mathcal{A}}_0 - i[\vec{A}\,\vec{\mathcal{A}}_0 - \vec{\mathcal{A}}_0\vec{A}] = \\ &\left[\frac{r^2R''_A - 2T_A(T_A R_A + 2U_A S_A) - 2U_A(U_A R_A + 2U_A S_A)}{r^4}\right]\hat{\alpha} \\ &+ \left[\frac{r^2 S_A - 3T_A(U_A R_A + 2T_A S_A) - 3U_A(T_A R_A + 2S_A U_A)}{r^5}\right]\hat{\beta}\end{aligned} \quad (68)$$

$$\begin{aligned}&\nabla(\vec{D}^1\phi_e) + i\left[\vec{A}, \vec{D}^1\phi_e\right] = \\ &\left[\frac{r^2 N''_{\phi_e} - 2T_A(T_A N_{\phi_e} + 2U_A M_{\phi_e}) - 2U_A(U_A N_{\phi_e} + 2T_A M_{\phi_e})}{r^4}\right]\hat{\alpha} \\ &+ \left[\frac{r^2 M''_{\phi_e} - 3T_A(U_A N_{\phi_e} + 2T_A M_{\phi_e}) - 3U_A(T_A N_{\phi_e} + 2U_A M_{\phi_e})}{r^5}\right]\hat{\beta}\end{aligned} \quad (69)$$

Equation (61) is satisfied if the coefficients of $\vec{T}_A, \vec{U}_A, \hat{\alpha}$ and $\hat{\beta}$ are zero that gives the system of nonlinear differential equations

$$\begin{aligned}&r^2 T''_A - T_A(T_A{}^2 + 7U_A{}^2 - 1) + T_A(R_A{}^2 + 4S_A{}^2) \\ &+ 4R_A U_A S_A - T_A(N_{\phi_e}{}^2 + 4M_{\phi_e}{}^2) - 4U_A N_{\phi_e} M_{\phi_e} = 0\end{aligned} \quad (70)$$

$$\begin{aligned} r^2U''_A - U_A\left(7T_A{}^2 + U_A{}^2 - 1\right) + U_A\left(R_A{}^2 + 4S_A{}^2\right) \\ + 4T_AR_AS_A - U_A\left(N_{\phi_e}{}^2 + 4M_{\phi_e}{}^2\right) - 4T_AN_{\phi_e}M_{\phi_e} = 0 \end{aligned} \quad (71)$$

$$r^2R''_A - 2R_A\left(T_A{}^2 + U_A{}^2\right) - 8T_AU_AS_A = 0 \quad (72)$$

$$r^2S_A - 2S_A\left(T_A{}^2 + U_A{}^2\right) - 6T_AU_AR_A = 0 \quad (73)$$

$$r^2N''_{\phi_e} - 2N_{\phi_e}\left(T_A{}^2 + U_A{}^2\right) - 8T_AU_AM_{\phi_e} = 0 \quad (74)$$

$$r^2M''_{\phi_e} - 6M_{\phi_e}\left(T_A{}^2 + U_A{}^2\right) - 6T_AU_AN_{\phi_e} = 0 \quad (75)$$

Similarly eqs (62),(64) and (66) give the system of nonlinear differential equations

$$\begin{aligned} r^2T''_B - T_B\left(T_B{}^2 + 7U_B{}^2 - 1\right) + T_B\left(R_B{}^2 + 4S_B{}^2\right) \\ + 4R_BU_BS_B - T_B\left(N_{\phi_g}{}^2 + 4M_{\phi_g}{}^2\right) - 4U_BN_{\phi_g}M_{\phi_g} = 0 \end{aligned} \quad (76)$$

$$\begin{aligned} r^2U''_B - U_B\left(7T_B{}^2 + U_B{}^2 - 1\right) + U_B\left(R_B{}^2 + 4S_B{}^2\right) \\ + 4T_BR_BS_B - U_B\left(N_{\phi_g}{}^2 + 4M_{\phi_g}{}^2\right) - 4T_BN_{\phi_g}M_{\phi_g} = 0 \end{aligned} \quad (77)$$

$$r^2R''_B - 2R_B\left(T_B{}^2 + U_B{}^2\right) - 8T_BU_BS_B = 0 \quad (78)$$

$$r^2S''_B - 2S_B\left(T_B{}^2 + U_B{}^2\right) - 6T_BU_BR_B = 0 \quad (79)$$

$$r^2N''_{\phi_g} - 2N_{\phi_g}\left(T_B{}^2 + U_B{}^2\right) - 8T_BU_BM_{\phi_g} = 0 \quad (80)$$

$$r^2M''_{\phi_g} - 6M_{\phi_g}\left(T_B{}^2 + U_B{}^2\right) - 6T_BU_BN_{\phi_g} = 0 \quad (81)$$

above system of second order non-linear differential equations (70) to(81) belong to the non-temporal gauge conditions $A_0^a \neq 0$ and $B_0^a \neq 0$ and the energy for this case is calculated by using the energy- momentum tensor $T^{\mu\nu}$ [4] as

$$m = \int d^3\vec{x}\, T^{00} = \int d^3\vec{x}\left[G_{0i}^a \times G_{0i}^a + \tilde{G}_{0i}^a \times \tilde{G}_{0i}^a + D_0\phi^a \times D_0\phi^a - \mathcal{L}\right] (82)$$

Using eqs. (9) and (10) the above expression yields

$$\begin{aligned} m = \int d^3\vec{x}\left[\frac{1}{2}A_{0i}^a\, A_{0i}^a + \frac{1}{2}A_{jk}^a\, A_{jk}^a + \frac{1}{2}D_i^1\phi_e^a\, D_i^1\phi_e^a - V\left(\phi_e^a\right)\right] \\ + \int d^3\vec{x}\left[\frac{15}{16}B_{0i}^a\, B_{0i}^a + \frac{3}{8}B_{jk}^a\, B_{jk}^a + \frac{1}{2}D_i^2\phi_g^a\, D_i^2\phi_g^a - V\left(\phi_g^a\right)\right] \end{aligned} \quad (83))$$

Using eqn.(25)[31] we get

$$\begin{aligned} m = \frac{1}{e^2}\mathrm{Tr}\int d^3\vec{x}\left[A_{0i}\, A_{0i} + A_{jk}\, A_{jk} + D_i^1\phi_e\, D_i^1\phi_e\right] \\ + \frac{1}{g^2}\mathrm{Tr}\int d^3\vec{x}\left[\frac{15}{8}B_{0i}\, B_{0i} + \frac{3}{4}B_{jk}\, B_{jk} + D_i^2\phi_g\, D_i^2\phi_g\right] \end{aligned}$$

$$\begin{aligned} = \frac{4\pi}{e^2}\int_0^\infty dr\, 4\Bigg\{ & \left[2\left(T_A'^2 + U_A'^2\right) + \frac{\left(1 - T_A{}^2 - U_A{}^2\right)}{r^2} + \frac{12T_A{}^2U_A{}^2}{r^2}\right] \\ & 2\left[\frac{\left(rR'_A - R_A\right)^2}{r^2} + \frac{4}{3}\frac{\left(rS'_A - S_A\right)^2}{r^2} + \frac{2\left(T_AR_A + 2U_AS_A\right)^2 + \left(U_AR_A + 2T_AS_A\right)^2}{r^2}\right] \\ & + 2\left[\frac{\left(rN'_{\phi_e} - N_{\phi_e}\right)^2}{r^2} + \frac{4}{3}\frac{\left(rM'_{\phi_e} - M_{\phi_e}\right)^2}{r^2} + \frac{2\left(T_AN_{\phi_e} + 2U_AM_{\phi_e}\right)^2 + \left(U_AN_{\phi_e} + 2T_AM_{\phi_e}\right)^2}{r^2}\right]\Bigg\} \end{aligned}$$

$$\begin{aligned} + \frac{4\pi}{g^2}\int_0^\infty dr \Bigg\{ & 4.\frac{3}{4}\left[2\left(T_B'^2 + U_B'^2\right) + \frac{\left(1 - T_B{}^2 - U_B{}^2\right)}{r^2} + \frac{12T_B{}^2U_B{}^2}{r^2}\right] \\ & + 2.\frac{15}{4}\left[\frac{\left(rR'_B - R_B\right)^2}{r^2} + \frac{4}{3}\frac{\left(rS'_B - S_B\right)^2}{r^2} + \frac{2\left(T_BR_B + 2U_BS_B\right)^2 + \left(U_BR_B + 2T_BS_B\right)^2}{r^2}\right] \\ & + 2\left[\frac{\left(rN'_{\phi_g} - N_{\phi_g}\right)^2}{r^2} + \frac{4}{3}\frac{\left(rM'_{\phi_g} - M_{\phi_g}\right)^2}{r^2} + \frac{2\left(T_BN_{\phi_g} + 2U_BM_{\phi_g}\right)^2 + \left(U_BN_{\phi_g} + 2T_BM_{\phi_g}\right)^2}{r^2}\right]\Bigg\} \end{aligned}$$

(84)

thus our system in the gauge $A_0^a \neq 0$ and $B_0^a \neq 0$, is now described by the second order non-linear differential equations (70) to (81) and the energy of this system is expressed by eqn (95) . However, the energy diverges at $r \to 0$. Therefore , to avoid the singularity at $r \to 0$, we impose following boundary conditions ……..In order to avoid the terms becoming singular as $r \to 0$, the following boundary conditions are required to be obeyed

$$\begin{aligned} & U_A(0) = \pm 1 = U_B(0) \\ & T_A \xrightarrow[r \to 0]{} \xi r^{1+\eta}, \quad T_B \xrightarrow[r \to 0]{} \xi_1 r^{1+\eta_1} \end{aligned} \quad (86a)$$

or

$$\begin{aligned} & U_A \xrightarrow[r \to 0]{} \xi r^{1+\eta}, \quad U_B \xrightarrow[r \to 0]{} \xi_1 r^{1+\eta_1} \\ & T_A(0) = \pm 1 = T_B(0) \end{aligned} \quad (86b)$$

where $\xi, \xi_1$ and $\eta, \eta_1$ are constants with $\eta, \eta_1 \geq 0$. Thus the energy (85) becomes finite when its parameters obey the boundary conditions (86). When these boundary conditions are obeyed by the solutions of eqs (70) to (81), the same would be the $A_0^a \neq 0$ and $B_0^a \neq 0$, the finite energy solutions. Thus our aim now is to obtain the solutions of second order differential eqs (70) to (81) which obey eqn.(86). For the above purpose if we let us put $T_A = T_B = F_A = F_B = M_{\phi_e} = M_{\phi_g} = 0$, we find that out of the twelve equations only the following six remain

$$r^2 U_A'' - U_A\left(U_A{}^2 + N_{\phi_e}{}^2 - R_A{}^2 - 1\right) = 0 \qquad (87)$$

$$r^2 R_A'' - 2R_A U_A{}^2 = 0 \qquad (88)$$

$$r^2 N_{\phi_e}'' - 2N_{\phi_e} U_A{}^2 = 0 \qquad (89)$$

$$r^2 U_B'' - U_B\left(U_B{}^2 + N_{\phi_g}{}^2 - R_B{}^2 - 1\right) = 0 \qquad (90)$$

$$r^2 R_B'' - 2R_B U_B{}^2 = 0 \qquad (91)$$

$$r^2 N_{\phi_g}'' - 2N_{\phi_g} U_B{}^2 = 0 \qquad (92)$$

It is interesting to note that the first three equation (87), (88) and (89) exactly match the Prasad and Sommerfield equations of motion.[32] Similar matching exists for the eqs. (90) (91) and (92) as well and the solutions of these equation comes out as [32]

$$U_A = \frac{\beta_A r}{\sinh \beta_A r} \qquad (93)$$

$$N_{\phi_e} = \pm \cosh \theta_A\left(\beta_A r \coth \beta_A r - 1\right) \qquad (94)$$

$$R_A = \pm \sinh \theta_A\left(\beta_A r \coth \beta_A r - 1\right) \qquad (95)$$

$$U_B = \frac{\beta_B r}{\sinh \beta_B r} \qquad (96)$$

$$N_{\phi_g} = \pm \cosh \theta_B\left(\beta_B r \coth \beta_B r - 1\right) \qquad (97)$$

$$R_B = \pm \sinh \theta_B\left(\beta_B r \coth \beta_B r - 1\right) \qquad (98)$$

$$T_A = T_B = M_{\phi_e} = M_{\phi_g} = S_A = S_B = 0 \qquad (99)$$

where $\beta_A \beta_B$ $\theta_A$ and $\theta_B$ are arbitrary constants. The finite energy corresponding to these solutions is obtained by substituting (93) to (98) above solutions when put in (85) give

$$m = \frac{4\pi}{e^2}\int_0^\infty 4\frac{\left(U_A N_{\phi_e}\right)^2}{r^2}dr + \frac{4\pi}{g^2}\int_0^\infty 4\frac{\left(U_B N_{\phi_g}\right)^2}{r^2}dr$$
$$= \frac{16\pi}{e^2}\beta_A \cosh^2\theta_A + \frac{16\pi}{g^2}\beta_B \cosh^2\theta_B \qquad (100)$$

## 4. Electric and Magnetic Charge

In order to show that the obtained solutions (93) to (99) having the finite energy (100), are dyon solutions in non- temporal gauge, we shall calculate the electric and magnetic charges. For that purpose we introduce the unit vectors $\hat{\phi}_e^a$ and $\hat{\phi}_g^a$ defined by[28]

$$\hat{\phi}_{e,g}^a = \frac{\phi_{e,g}^a}{\left(\phi_{e,g}^a\,\phi_{e,g}^a\right)^{1/2}} = \frac{\hat{\alpha}^a}{2r} \qquad (101)$$

From previous paper [31]

$$\frac{1}{2}\varepsilon_{ijk}A^{jka} = \frac{1}{e}\mathcal{A}_i^a \qquad (102a)$$

and $$\frac{1}{2}\varepsilon_{ijk}B^{jka} = \frac{1}{g}\tilde{\mathcal{B}}_i^a \qquad (102b)$$

We introduce the field [22]

$$A_{0i}^a = \frac{1}{e}\mathcal{A}_0^a \qquad (103a)$$

and $$B_{0i}^a = \frac{1}{g}\tilde{\mathcal{B}}_i^a \qquad (103b)$$

The electric charge $q_e$ may now be calculated by using eqs. (102b) and (103a) ) as

$$q_e = \frac{1}{4\pi}\int \hat{\phi}_e^a G_{0i}^a ds_i \qquad (93) \qquad (104)$$

where $G_{0i}$ can be had from eqn (2) and $ds_i$ denote the surface element of the surface at infinity which is also the boundary of the static fields. (94)

(95)

$$q_g = \frac{1}{4\pi}\int \hat{\phi}_e^a[A_{oi}^a + \frac{1}{2}\varepsilon_{ijk}B^{jka}]ds_i$$
$$= \frac{1}{4\pi}\int \hat{\phi}_e^a \frac{\mathcal{A}_0^a}{e}ds_i + \frac{1}{4\pi}\int \hat{\phi}_e^a \frac{\tilde{\mathcal{B}}_i^a}{g}ds_i$$
$$\alpha^a\alpha^a = 4r^2$$
$$= \frac{1}{8\pi}\int \frac{\hat{\alpha}^a}{r}\frac{\tilde{R}_{0A}x_i\hat{\alpha}^a}{e}ds_i + \frac{1}{8\pi}\int \frac{\hat{\alpha}^a}{r}\frac{\tilde{R}_B x_i\hat{\alpha}^a}{g}ds_i$$
$$= \frac{1}{2\pi e}\int \frac{rR_A' - R_A}{r^3}\vec{x}\cdot ds + \frac{1}{2\pi g}\int \frac{(1-U_B^2)}{r}\vec{x}\cdot ds$$
$$= \pm\frac{2\sinh\theta_A}{e} + \frac{2}{g} \qquad (105)$$

The magnetic charge likewise is obtained

$$q_g = \frac{1}{4\pi}\int \hat{\phi}_g^a \frac{1}{2}\varepsilon_{ijk}G^{jka}ds_i$$
$$= \frac{1}{4\pi}\int \hat{\phi}_g^a[\frac{1}{2}\varepsilon_{ijk}A^{jka} + \frac{3}{2}B_{oi}^a]ds_i$$
$$= \frac{1}{4\pi}\int \hat{\phi}_g^a \frac{\mathcal{A}_i^a}{e}ds_i + \frac{3}{8\pi}\int \hat{\phi}_g^a \frac{\tilde{\mathcal{B}}_0^a}{g}ds_i$$

$$=+\frac{1}{2\pi e}\int\frac{(1-U_A^2)}{r}\vec{x}\cdot\vec{ds}+\frac{3}{4\pi g}\int\frac{rR'_B-R_B}{r^3}\vec{x}\cdot\vec{ds}$$
$$=\frac{2}{e}\pm\frac{3\sinh\theta_B}{4g} \qquad (106)$$

Thus the obtained solutions are dyonic solutions in temporal gauge with electric charge $\pm\frac{2\sinh\theta_A}{e}+\frac{2}{g}$ of and magnetic charge of $\frac{2}{e}\pm\frac{3\sinh\theta_B}{4g}$.

**5. Adjoining Solutions**

Equations (93) to (98) provide the non-temporal gauge solutions in which both $A_0^a$ and $B_0^a$ were non-vanishing. However, remaining in the realm of non-temporal gauge conditions we can have particular case of (a) $A_0^a=0$ and $B_0^a\neq 0$ and (b) $A_0^a\neq 0$ and $B_0^a=0$. Adopting the procedure of previous sections, we show in the following that in these particular cases of temporal gauge too, the obtained solutions though are finite energy dyonic but are different from the previous section.

**Case (a)** $A_0^a=0,\ B_0^a\neq 0$

The vanishing of $A_0^a$ implies the vanishing of $R_A$ and $S_A$, accordingly the field equations (87)-(75) become

$$r^2U''_A-U_A\left(U_A{}^2+N_{\phi_e}{}^2-1\right)=0 \qquad (107)$$

$$r^2N''_{\phi_e}-2N_{\phi_e}U_A{}^2=0 \qquad (108)$$

rest eqs.(91) – (94) remain same.

Since, the second order differential equations 107) to (108) again match with those of Prasad and Sommerfield[32], the solutions in this case of $A_0^a=0,\ B_0^a\neq 0$ are the solutions of field equations (107), (108) and (90)-(92) which are written as

$$U_A=\frac{\beta_A r}{\sinh\beta_A r} \qquad (109)$$

$$N_{\phi_e}=\pm(\beta_A r\coth\beta_A r-1) \qquad (110)$$

$$U_B=\frac{\beta_B r}{\sinh\beta_B r} \qquad (111)$$

$$N_{\phi_g}=\pm\cosh\theta_B(\beta_B r\coth\beta_B r-1) \qquad (112)$$

$$R_B=\pm\sinh\theta_B(\beta_B r\coth\beta_B r-1) \qquad (113)$$

The energy of these solutions can also be calculated from eqn. (85) by putting $R_A=0=S_A$, which will also admit the boundary conditions (86) resulting in the following expression of energy that would be finite

$$m=\frac{1}{e^2}\mathrm{Tr}\int d^3\vec{x}\left[2A_{jk}\,A_{jk}\ \right]$$
$$+\frac{1}{g^2}\mathrm{Tr}\int d^3\vec{x}\left[\frac{15}{8}B_{0i}\,B_{0i}+\frac{3}{4}B_{jk}\,B_{jk}+D_i^2\phi_g\,D_i^2\phi_g\ \right] \qquad (114)$$
$$=\frac{32\pi}{e^2}\beta_A+\frac{16\pi}{g^2}\beta_B\cosh^2\theta_B$$

The electric and magnetic charges for this case can also be calculated as

$$q_e\ =\frac{2}{g} \qquad (115a)$$

$$q_g\ =\frac{2}{e}\pm\frac{3\sinh\theta_B}{4g} \qquad (115b)$$

Thus in the gauge $A_0^a=0,\ B_0^a\neq 0$, we have finite energy dyon solutions (109)-(113) with finite energy (114) and dyon charges (115).

**Case (b)** $A_0^a\neq 0,\ B_0^a=0$

In this case $R_B$ and $S_B$ vanish. The field equation (87)-(89) remain same, whereas eqn. (90)-(92) after substituting $R_B=0=S_B$ reduce to following two equations

$$r^2U''_B-U_B\left(U_B{}^2+N_{\phi_g}{}^2-1\right)=0 \qquad (116)$$

$$r^2N''_{\phi_g}-2N_{\phi_g}U_B{}^2=0 \qquad (117)$$

Corresponding to $A_0^a\neq 0$, we shall have three equations viz. (87), (88) and (89). The solutions of these five equations yields

$$U_A=\frac{\beta_A r}{\sinh\beta_A r} \qquad (118)$$

(109)

$$N_{\phi_e}=\pm\cosh\theta_A(\beta_A r\coth\beta_A r-1) \qquad (119)$$

(110)

$$R_A=\pm\sinh\theta_A(\beta_A r\coth\beta_A r-1) \qquad (120)$$

(111)

$$U_B=\frac{\beta_B r}{\sinh\beta_B r} \qquad (121)$$

$$N_{\phi_g}=\pm(\beta_B r\coth\beta_B r-1) \qquad (122)$$

where equations (121) and (122) are the Prasad-Sommerfield[32] solutions of equations (116) and (117) and the first three (118-120) are the solutions of equations (87), (88) and (89).

The finite energy for this case also from eqn. (85) on substituting equation (99) and $R_B = 0$ and accommodating the boundary conditions (86) becomes

$$m = \frac{16\pi}{e^2}\beta_A \cosh^2\theta_A + \frac{32\pi}{g^2}\beta_B \tag{123}$$

The electric and magnetic charges for this case are

$$q_e = \pm\frac{2\sinh\theta_A}{e} + \frac{2}{g} \tag{124a}$$

$$q_g = \frac{2}{e} \tag{124b}$$

**Conclusion**

Using a Cabbibo-Ferrari type non-Abelian field tensor, the dyon-solutions have been obtained in the temporal gauge. Introducing the quantities $\hat{\alpha}$ and $\hat{\beta}$ in terms of Gell-Mann matrices, three-vectors $\vec{P}, \vec{Q}, \vec{R}, \vec{S}, \vec{T}$ and $\vec{U}$ have been defined. The gauge fields have then been expressed in terms of these three-vectors which results in the reduction of second order non-linear field equations into the first order non-linear equations whose solutions employing the self-duality conditions lead to Euclidean space dyon solutions whose energy has been shown to be finite. The distinguishing feature of the obtained solutions is the use of Cabbibo-Ferrari type non-Abelian field tensor and the temporal gauge.

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